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Mean Queue Length of a Four-Server Feedback Queuing Model with Customer Returns to Any Server up to Finite Times

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ABSTRACT
A feedback queueing model that considers four servers—one of which is centrally connected to the other three—is presented in this paper.
Only the first server allows customers to access the system; they can then move on to the second, third, or fourth server. If the service does not satisfy the customer's needs, they may return to the previously visited server a limited number of times. The probabilities of going back to the servers is thought to be different with every visit.
The steady state equations have been used to calculate the mean queue lengths of the system. Using both numerical and graphical methods, the variations in the average mean queue lengths of the system have been identified.

Key Words: Feedback queuing system, Four types of servers, Revisit to servers.

1 Introduction

The literature on feedback queueing models is extensive in the field of queueing theory. Multi-server and multiclass queuing models have been discussed by a good number of researchers such as Jianghua and Jinting (2006), Houdt et al. (2008), Goswami and Pandit (2011), Sundri and Srinivasan (2012), Aristotles and Endah (2013), Ibe and Isijola (2014), Morozov (2014), Kang (2015), Yanfeng and Christos (2015), Zadeh (2015), Atar and Mendelson (2016), Avrachenkov et al. (2016), Jansen et al. (2016), Reed and Zhang (2017), Antonioli et al. (2018), Ginting et al. (2018).

Kumar and Taneja (2019), worked on the feedback queuing system comprising of three servers, in which one is centrally connected with the other two servers. Kamal et al. (2023) worked on four servers feedback queueing system in which a customer from outside can enter the system through first server only. After that she/he may go to any of the other three servers as per her/his need for service. If she/he is not satisfied with the service, revisit with different probability is also possible. But they assumed the revisit only once. There may be systems in place where services are provided in such a manner having four servers with the provision of service more than twice; as a result, Nidhi et al. (2024) dedicated to examining these systems. They assumed the revisits more than once but up to finite number of times. They discussed a queueing system where customers can either proceed to the second, third or fourth server based on their pleasure with the service. There's also a possibility that she/he will quit the system after satisfaction or come back to the original server with criticism. In this instance, we have taken into account the circumstances in which a client is obligated to return up to a certain number of times. Each time you come back, your probability of getting on any given server are considered to be unique. Administrative offices, medical facilities and other organizations may all encounter this kind of circumstances. With the

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use of the differential-difference method, the queue lengths have been established by them. But they did not justify the findings numerically and graphically.

In this study, we have provided a numerical and graphical explanation of the model's results by giving different variables and queueing characteristics arbitrary numerical values in the derived formulae.

2 Notation

λ: Mean Arrival rate at 1st server (S_1)
μ_1: service rate of 1st server (S_1)
μ_2: service rate of 2ndserver (S_2)
μ_3: service rate of 3rd server (S_3)
μ_4: service rate of 4th server (S_4)
p^i_{12}: the probability of customer going from 1stto 2nd server ith time.
p^i_{13}: the probability of customer going from 1stto 3rd server ith time.
p^i_{14}: the probability of customer going from 1stto 4th server ith time.
p^i_2: the probability of exit of customer from 2nd server ith time.
p^i_{23}: the probability of customer going from 2nd to 3rd server ith time.
p^i_{24}: the probability of customer going from 2nd to 4th server ith time.
p^i_{21}: the probability of customer going from 2nd to 1st server ith time.
p^i_3: the probability of exit of customer from 3rd server ith time.
p^i_{31}: the probability of customer going from 3rd to 1st server ith time.
p^i_{32}: the probability of customer going from 3rd to 2nd server ith time.
p^i_{34}: the probability of customer going from 3rd to 4th server ith time
p^i_4: the probability of exit of customer from 4th server ith time.
p^i_{41} : the probability of exit of customer from 4th to 1st server ith time.
p^i_{42}: the probability of customer going from 4th to 2nd server ith time.
p^i_{43}: the probability of customer going from 4th to 3rd server ith time.

$$A_{12} = \sum_{i=1}^{n} a^i p^i_{12} \qquad A_{13} = \sum_{i=1}^{n} a^i p^i_{13}$$
$$A_{14} = \sum_{i=1}^{n} a^i p^i_{14}$$
$$B_2 = \sum_{i=1}^{n} b^i p^i_2,\ B_{21} = \sum_{i=1}^{n-1} b^i p^i_{21},\ B_{23} = \sum_{i=1}^{n} b^i p^i_{23},\ B_{24} = \sum_{i=1}^{n} b^i p^i_{24}$$
$$C_3 = \sum_{i=1}^{n} c^i p^i_3,\ C_{34} = \sum_{i=1}^{n} c^i p^i_{34},\ C_{31} = \sum_{i=1}^{n-1} c^i p^i_{31},\ C_{32} = \sum_{i=1}^{n-1} c^i p^i_{32}$$
$$D_4 = \sum_{i=1}^{n} d^i p^i_4,\ D_{43} = \sum_{i=1}^{n-1} d^i p^i_{43},\ D_{42} = \sum_{i=1}^{n-1} d^i p^i_{42},\ D_{41} = \sum_{i=1}^{n-1} d^i p^i_{41}$$

3 Formulation of Problem

According to Nidhi et al. (2024), the queue network consists of four service channels in such a manner that first server (S_1) is centrally linked with the remaining three parallel servers (S_2), (S_3) and (S_4). It is assumed that customer arrives at first server (S_1) from outside the system and then may go to any one of the second (S_2), third (S_3) and fourth (S_4) server. The situation has been shown by the following sate transition diagram:

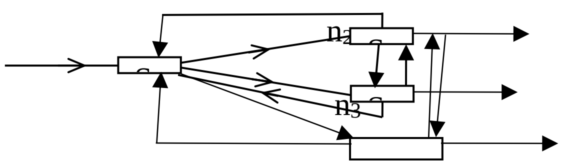

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Diagram Showing Movement of the Customers from Various Servers

A customer either goes to the second, third, or fourth server after receiving service from the first server ith time, so that $p^i_{12} + p^i_{13} + p^i_{14} = 1$. Once a customer is satisfied, they can leave the system from the second server or proceed to the third, or fourth server or back to first server so that $p^i_2 + p^i_{21} + p^i_{23} + p^i_{24} = 1$. In order to $p^i_3 + p^i_{31} + p^i_{32} + p^i_{34} = 1$, s/he can leave the system from the third server or go to the fourth, second, or first server. In a similar vein, the user may leave the system from the fourth server or switch to the third, second, or first server. Thus, $p^i_4 + p^i_{41} + p^i_{42} + p^i_{43} = 1$. As a result, we have:

$A_{12}+A_{13}+A_{14}=1$
$B_2+B_{21}+B_{23}+B_{24}=1$
$C_3+C_{31}+C_{32}+C_{34}=1$
$D_4+D_{41}+D_{42}+D_{43}=1$

Let Q_{n_1,n_2,n_3,n_4} is the probability of having n_1, n_2, n_3, n_4 customers at server 1st, 2nd, 3rd and 4th server at any time t then;

$$Q_{n_1 n_2 n_3 n_4} = \begin{cases} 1; n_1, n_2, n_3, n_4 \neq 0 \\ 0;\ otherwise \end{cases}$$

and

$$F(X,Y,Z,R) = \sum Q_{n_1 n_2 n_3 n_4} X^{n_1} Y^{n_2} Z^{n_3} R^{n_4}$$

where $|X| = |Y| = |Z| = |R| = 1$ … (1)

They further defined:

$$G_{n_2,n_3,n_4}(X) = \sum_{n_1=0}^{\infty} Q_{n_1 n_2 n_3 n_4} X^{n_1} \quad \ldots (2)$$

$$G_{n_3,n_4}(X,Y) = \sum_{n_2=0}^{\infty} G_{n_2,n_3,n_4}(X)\, Y^{n_2} \quad \ldots (3)$$

$$G_{n_4}(X,Y,Z) = \sum_{n_3=0}^{\infty} G_{n_3,n_4}(X,Y) Z^{n_3} \quad \ldots (4)$$

$$F(X,Y,Z,R) = \sum_{n_4=0}^{\infty} G_{n_4}(X,Y,Z) R^{n_4} \quad \ldots (5)$$

For convenience,

$G_0(Y, Z, R) = G_1$
$G_0(X, Z, R) = G_2$
$G_0(X, Y, R) = G_3$
$G_0(X, Y, Z) = G_4$

They obtained the following by solving the queueing system's steady state equations:

$$G_1 = \frac{\begin{array}{l} \mu_1 B_{21}(A_{12}(1-C_{34}B_{43}) + C_{32}(A_{14}D_{43} + A_{13}) + (A_{13}C_{34} + A_{14})D_{42}) \\ + \mu_1(C_{32}(B_{23}D_{43} + B_{23}) + B_{23}(C_{34}+1)D_{42}) + \mu_1 C_{31}(A_{12}(B_{23}D_{43} + B_{23}) \\ + (A_{14} - A_{13})B_{23}D_{42}) + \lambda(C_{32}(-B_{23}D_{43} - B_{23}) + B_{23}(-C_{34} - 1)D_{42}) \\ + \mu_1(A_{12}B_{23}(C_{34}+1) + (A_{13} - A_{14})B_{23}C_{32})D_{41} \end{array}}{\begin{array}{l} \mu_1 B_{21}(A_{12}(1-C_{34}B_{43}) + C_{32}(A_{14}D_{43} + A_{13}) + (A_{13}C_{34} + A_{14})D_{42}) \\ + \mu_1(C_{32}(B_{23}D_{43} + B_{23}) + B_{23}(C_{34}+1)D_{42}) + \mu_1 C_{31}(A_{12}(B_{23}D_{43} + B_{23}) \\ + (A_{14} - A_{13})B_{23}D_{42}) + \mu_1(A_{12}B_{23}(C_{34}+1) + (A_{13} - A_{4})B_{23}C_{32})D_{41} \end{array}} \quad \ldots (6)$$

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$$G_2 = \frac{\begin{array}{l}\mu_2 B_{21}(A_{12}(1-C_{34}D_{43})+C_{32}(A_{14}D_{43}+A_{13})+(A_{13}C_{34}+A_{14})D_{42}) \\ +\mu_3(A_{12}(1-C_{34}D_{43})+C_{32}(A_{14}D_{43}+A_{13})+(A_{13}C_{34}+A_{14})D_{42} \\ +\mu_2(C_{32}(B_{23}D_{43}+B_{23})+B_{23}(C_{34}+1)D_{42})+\mu_2 C_{31}(A_{12}(B_{23}D_{43}+B_{23}) \\ +(A_{14}-A_{13})B_{23}D_{42})+\mu_2(A_{12}B_{23}(C_{34}+1)+(A_{13}-A_{14})B_{23}C_{32})D_{41}\end{array}}{\begin{array}{l}\mu_2 B_{21}(A_{12}(1-C_{34}D_{43})+C_{32}(A_{14}D_{43}+A_{13})+(A_{13}C_{34}+A_{14})D_{42}) \\ +\mu_2(C_{32}(B_{23}D_{43}+B_{23})+B_{23}(C_{34}+1)D_{42})+\mu_2 C_{31}(A_{12}(B_{23}D_{43}+B_{23}) \\ +(A_{14}-A_{13})(B_{23}D_{42})+\mu_2(A_{12}B_{23}(C_{34}+1)+(A_{13}-A_{14})B_{23}C_{32})D_{41}\end{array}} \quad \ldots (7)$$

$$G_3 = \frac{\begin{array}{l}\mu_3 B_{21}(A_{12}(1-C_{34}D_{43})+C_{32}(A_{14}D_{43}+A_{13})+(A_{13}C_{34}+A_{14})D_{42}) \\ +\mu_3(C_{32}(B_{23}D_{43}+B_{23})+B_{23}(C_{34}+1)D_{42})+\mu_3 C_{31}(A_{12}(B_{23}D_{43}+B_{23}) \\ +(A_{14}-A_{13})B_{23}D_{42})+\lambda(A_{12}B_{23}D_{43}+B_{23})+(A_{14}-A_{13})B_{23}D_{42}) \\ +\mu_3(A_{12}B_{23}(C_{34}+1)+(A_{13}-A_{14})B_{23}C_{32})D_{41}\end{array}}{\begin{array}{l}\mu_2 B_{21}(A_{12}(1-C_{34}D_{43})+C_{32}(A_{14}D_{43}+A_{13})+(A_{13}C_{34}+A_{14})D_{42}) \\ +\mu_3(C_{32}(B_{23}D_{43}+B_{23})+B_{23}(C_{34}+1)D_{42})+\mu_3 C_{31}(A_{12}(B_{23}D_{43}+B_{23}) \\ +(A_{14}-A_{13})(B_{23}D_{42})+\mu_3(A_{12}B_{23}(C_{34}+1)+(A_{13}-A_{14})B_{23}C_{32})D_{41}\end{array}} \quad \ldots (8)$$

$$G_4 = \frac{\begin{array}{l}\mu_4 B_{21}(A_{12}(1-C_{34}D_{43})+C_{32}(A_{14}D_{43}+A_{13})+(A_{13}C_{34}+A_{14})D_{42}) \\ +\mu_4(C_{32}(B_{23}D_{43}+B_{23})+B_{23}(C_{34}+1)D_{42})+\mu_4 C_{31}(A_{12}(B_{23}D_{43}+B_{23}) \\ +(A_{14}-A_{13})B_{23}D_{42})+\mu_4(A_{12}B_{23}(C_{34}+1)+(A_{14}-A_{13})B_{23}C_{32})D_{41} \\ +\lambda(A_{12}B_{23}(C_{34}+1)+(A_{13}-A_{14})B_{23}C_{32})\end{array}}{\begin{array}{l}\mu_4 B_{21}(A_{12}(1-C_{34}D_{43})+C_{32}(A_{14}D_{43}+A_{13})+(A_{13}C_{34}+A_{14})D_{42}) \\ +\mu_4(C_{32}(B_{23}D_{43}+B_{23})+B_{23}(C_{34}+1)D_{42})+\mu_4 C_{31}(A_{12}(B_{23}D_{43}+B_{23}) \\ +(A_{14}-A_{13})(B_{23}D_{42})+\mu_4(A_{12}B_{23}(C_{34}+1)+(A_{13}-A_{14})B_{23}C_{32})D_{41}\end{array}} \quad \ldots (9)$$

If Lq be the mean queue length of the whole system then:

$$Lq = -\mu_1\left[\frac{(\mu_1G_1-\mu_2B_{21}G_2-\mu_3C_{31}G_3-\mu_4D_{41}G_4)}{(-\lambda+\mu_1-\mu_2B_{21}-\mu_3C_{31}-\mu_4D_{41})^2}+\frac{G_1}{(-\lambda+\mu_1-\mu_2B_{21}-\mu_3C_{31}-\mu_4D_{41})}\right]$$
$$+-\mu_2\left[\frac{(-\mu_1A_2G_1+\mu_2G_2-\mu_3G_3C_{32}-\mu_4G_4D_{42})}{(-\mu_1A_{12}+\mu_2-\mu_3C_{32}-\mu_4D_{42})^2}+\frac{G_2}{(-\mu_1A_{12}+\mu_2-\mu_3C_{32}-\mu_4D_{42})}\right]$$
$$-\mu_3\left[\frac{(-\mu_1G_1A_{13}-\mu_2G_2B_{23}+\mu_3G_3-\mu_4G_4D_{43})}{(-\mu_1A_{13}-\mu_2B_{23}+\mu_3-\mu_4D_{43})^2}+\frac{G_3}{(-\mu_1A_{13}-\mu_2B_{23}+\mu_3-\mu_4D_{43})}\right]$$
$$+-\mu_4\left[\frac{(-\mu_1G_1A_{14}-\mu_2G_2B_{24}-\mu_3G_3C_{34}+\mu_4G_4)}{(-\mu_1A_{14}-\mu_2B_{24}-\mu_3C_{34}+\mu_4)^2}+\frac{G_4}{(-\mu_1A_{14}-\mu_2B_{24}-\mu_3C_{34}+\mu_4)}\right]$$

… (10)

4. Numerical Results and Discussion:

4.1 Behaviour of mean queue length of the system (L_q) with respect to arrival rate (λ) for different values of μ_1 is depicted in Table 4.1 and in Fig. 4.1 keeping the values of other parameters as fixed.

Table 4.1

μ_2 = 0.1, μ_3= 5, μ_4= 0.2, A_{14}=,0.5 A_{13}=0.3, A_{12}= 0.2, B_2=0.4, B_{21}=0.3, B_{23}=0.2, B_{24}=0.1, C_3=0.6, C_{32}=0.15, C_{31}=0.2, C_{34}=0.05, D_4=0.1, D_{41}=0.7, D_{42}=0.15, D_{43}=0.05			
	μ_1=0.5	**μ_1=1**	**μ_1=2**
λ	**Lq**	**Lq**	**Lq**
4	14.30324	11.2892	7.506586
5	14.78821	11.66644	6.663907
6	15.27107	12.08646	6.138637
7	15.73794	12.51168	5.734197
8	16.18082	12.92305	5.371759
9	16.59428	13.30926	5.012956
10	16.97416	13.66257	4.6361
11	17.31689	13.97704	4.227255
12	17.61917	14.24764	3.776382
13	17.87779	14.46975	3.275483
14	18.0895	14.6389	2.71762

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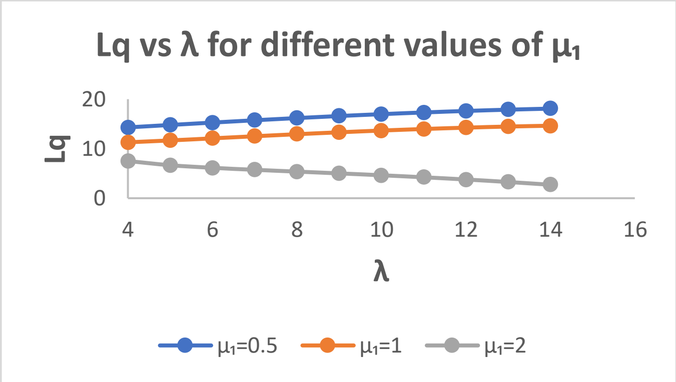

Fig. 4.1

Following can be interpreted from **Table 4.1** and **Fig. 4.1**:

(i) Mean queue length (L_q) increases with the increase in λ for $\mu_1 < 2$ but decreases for $\mu_1 \geq 2$.

(ii) Mean queue length (L_q) decreases with respect to the increase in first server service rate (μ_1).

4.2. Behaviour of mean queue length of the system (L_q) with respect to service rate (μ_2) for different values of arrival rate λ is depicted in Table 4.2 and in Fig. 4.2 keeping the values of other parameters as fixed.

Table 4.2

$\mu_1 = 6$, $\mu_3= 5$, $\mu_4= 0.2$, A_{14}=,0.5 A_{13}=0.3, A_{12}= 0.2, B_2=0.4, B_{21}=0.3, B_{23}=0.2, B_{24}=0.1, C_3=0.6, C_{32}=0.15, C_{31}=0.2, C_{34}=0.05, D_4=0.1, D_{41}=0.7, D_{42}=0.15, D_{43}=0.05			
	λ=0.1	λ=0.2	λ=0.3
μ_2	Lq	Lq	Lq
12	61.24455	66.44511	73.67342
12.1	65.88566	71.59288	79.58573
12.2	70.92091	77.19773	86.05975
12.3	76.39575	83.31495	93.16894
12.4	82.36227	90.00839	100.9992
12.5	88.8804	97.35205	109.6514
12.6	96.01939	105.4321	119.2448
12.7	103.8595	114.3493	129.9213
12.8	112.4943	124.2223	141.8506
12.9	122.0333	135.1913	155.2374
13	132.6053	147.423	170.3303

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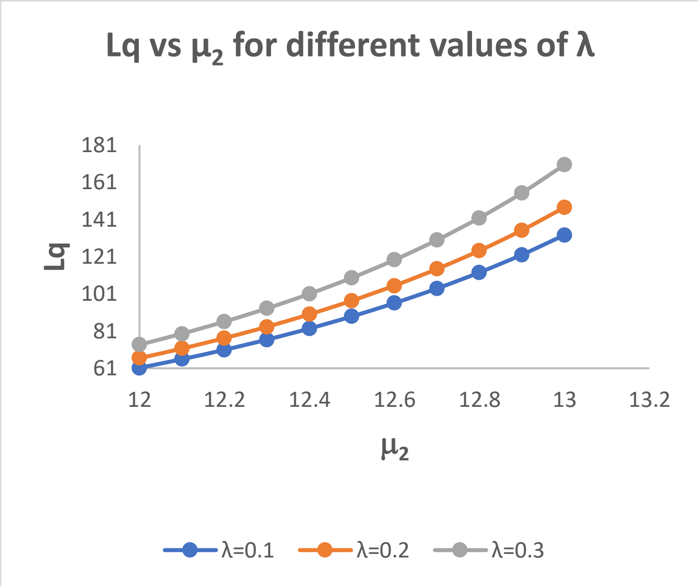

Fig. 4.2

Following can be interpreted from **Table 4.2** and **Fig. 4.2**:

(i) Mean queue length (L_q) increases with respect to the increase in second server service rate (μ_2).

(ii) Mean queue length (L_q) increases with the increase in λ.

4.3. Behaviour of mean queue length of the system (L_q) with respect to service rate of third server (μ_3) for different values of arrival rate λ is depicted in Table 4.3 and in Fig. 4.3 keeping the values of other parameters as fixed.

Table 4.3

$\mu_1 = 6$, $\mu_2= 1$, $\mu_4= 0.2$, $A_{14}=$,0.5 $A_{13}=0.3$, $A_{12}= 0.2$, $B_2=0.4$, $B_{21}=0.3$, $B_{23}=0.2$, $B_{24}=0.1$, $C_3=0.6$, $C_{32}=0.15$, $C_{31}=0.2$, $C_{34}=0.05$, $D_4=0.1$, $D_{41}=0.7$, $D_{42}=0.15$, $D_{43}=0.05$			
	$\lambda=0.1$	$\lambda=0.2$	$\lambda=0.3$
μ_3	Lq	Lq	Lq
12	7.980468	8.335539	8.73287
12.1	8.145444	8.51121	8.920744
12.2	8.312023	8.688806	9.110922
12.3	8.48029	8.868424	9.303517
12.4	8.650333	9.050163	9.498645
12.5	8.822241	9.234128	9.696426
12.6	8.996106	9.420424	9.896982
12.7	9.172022	9.609159	10.10044
12.8	9.350087	9.800446	10.30693
12.9	9.530399	9.994401	10.51659
13	9.713061	10.19114	10.72955

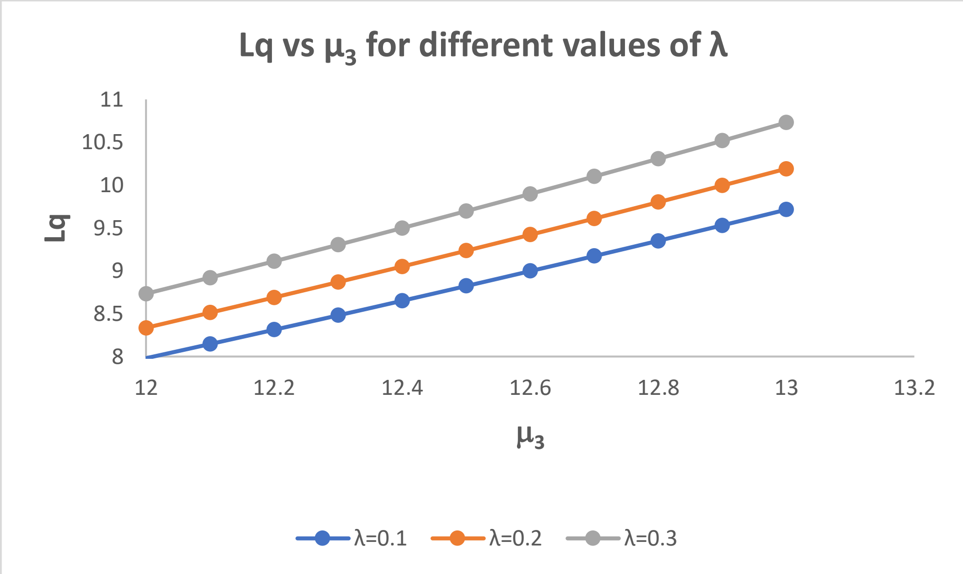

Fig. 4.3

Following can be interpreted from **Table 4.3** and **Fig. 4.3**:

(i) Mean queue length (L_q) increases with respect to the increase in third server service rate (μ_3).

(ii) Mean queue length (L_q) increases with the increase in λ.

4.4. Behaviour of mean queue length of the system (L_q) with respect to service rate of fourth server (μ_4) for different values of arrival rate λ is depicted in Table 4.4 and in Fig. 4.4 keeping the values of other parameters as fixed.

Table 4.4

$\mu_1 = 6$, $\mu_2= 1$, $\mu_3= 2$, $A_{14}=$,0.5 $A_{13}=0.3$, $A_{12}= 0.2$, $B_2=0.4$, $B_{21}=0.3$, $B_{23}=0.2$, $B_{24}=0.1$, $C_3=0.6$, $C_{31}=0.2$, $C_{32}=0.15$, $C_{34}=0.05$, $D_4=0.1$, $D_{41}=0.7$, $D_{42}=0.15$, $D_{43}=0.05$			
	λ=0.1	**λ=0.2**	**λ=0.3**
μ_4	**Lq**	**Lq**	**Lq**
12	10.10228	9.343082	8.589756
12.1	9.95728	9.216265	8.480692
12.2	9.817627	9.094074	8.375574
12.3	9.682975	8.976213	8.274147
12.4	9.553011	8.862407	8.176177
12.5	9.427447	8.752409	8.08145
12.6	9.30602	8.64599	7.98977
12.7	9.188489	8.542941	7.900958
12.8	9.074631	8.443069	7.814846
12.9	8.964243	8.346198	7.731284
13	8.857136	8.252163	7.650131

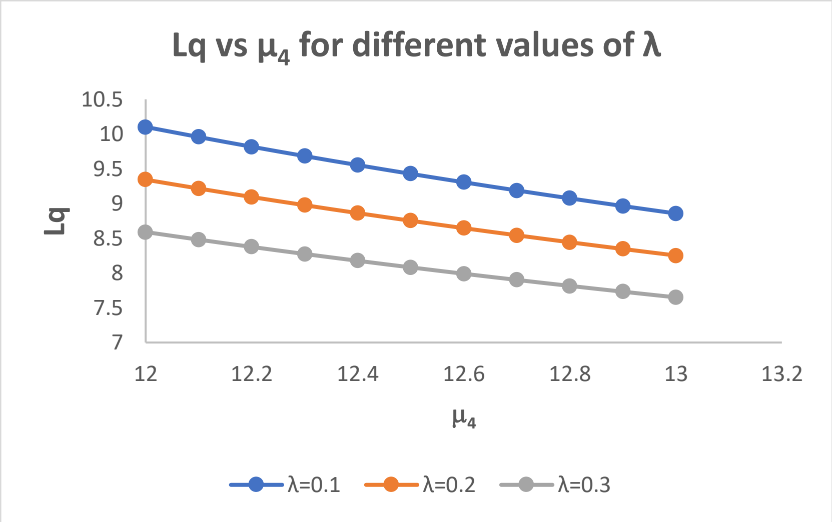

Fig. 4.4

Following can be interpreted from **Table 4.4** and **Fig. 4.4**:

(i) Mean queue length (L_q) decreases with respect to the increase in fourth server service rate (μ_4).

(ii) Mean queue length (L_q) decreases with the increase in λ.

4.5. Behaviour of mean queue length of the system (L_q) with respect to service rate of first server (μ_1) for different values of second server service rate (μ_2) is depicted in Table 4.5 and in Fig. 4.5 keeping the values of other parameters as fixed.

Table 4.5

$\lambda=0.1$, $\mu_3=2$, $\mu_4=5$, $A_{12}=0.2$, $A_{13}=0.3$, $A_{14}=0.5$, $B_2=0.4$, $B_{21}=0.3$, $B_{23}=0.2$, $B_{24}=0.1$, $C_3=0.6$, $C_{31}=0.2$, $C_{32}=0.15$, $C_{34}=0.05$, $D_4=0.1$, $D_{41}=0.7$, $D_{42}=0.15$, $D_{43}=0.05$			
	$\mu_2=6$	**$\mu_2=7$**	**$\mu_2=9$**
μ_1	**Lq**	**Lq**	**Lq**
4.4	10.05848	5.298269	0.246503
4.5	10.58011	5.633201	0.378574
4.6	11.45911	6.138671	0.563847
4.7	12.78912	6.850152	0.810396
4.8	14.71593	7.818046	1.128604
4.9	17.47127	9.114829	1.531892
5	21.43752	10.84682	2.037748
5.1	27.27925	13.17435	2.669202
5.2	36.23594	16.34792	3.456988
5.3	50.84967	20.77687	4.442787

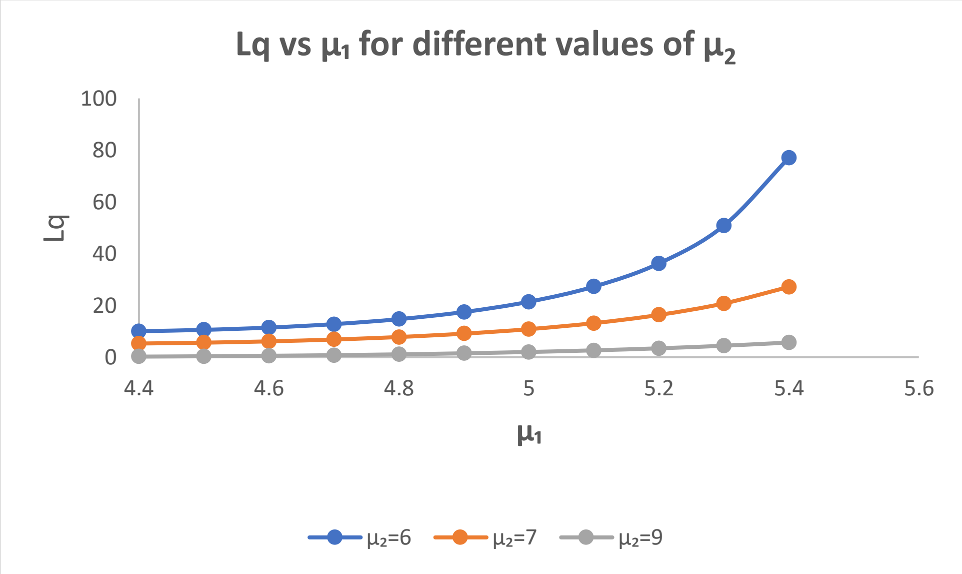

Fig. 4.5

Following can be interpreted from **Table 4.5** and **Fig. 4.5**:

(i) Mean queue length (L_q) decreases with respect to the increase in first server service rate (μ_1).

(ii) Mean queue length (L_q) decreases with the increase in second server service rate (μ_2).

4.6. Behaviour of mean queue length of the system (L_q) with respect to service rate of first server (μ_1) for different values of third server service rate (μ_3) is depicted in Table 4.6 and in Fig. 4.6 keeping the values of other parameters as fixed.

Table 4.6

λ=0.1, μ_2= 2, μ_4=8.5, A_{12}= 0.2, A_{13}=0.3, A_{14}=0.5, B_2=0.4, B_{21}=0.3, B_{23}=0.2, B_{24}=0.1, C_3=0.6, C_{31}=0.2, C_{32}=0.15, C_{34}=0.05, D_4=0.1, D_{41}=0.7, D_{42}=0.15, D_{43}=0.05			
	μ_3 = 16	**μ_3 = 17**	**μ_3 = 20**
μ_1	**Lq**	**Lq**	**Lq**
4	4.680863	5.523582	7.63165
4.1	4.908278	5.731876	7.798764
4.2	5.140963	5.945552	7.971221
4.3	5.37962	6.165234	8.149475
4.4	5.625002	6.39159	8.334016
4.5	5.877914	6.625334	8.525362
4.6	6.139225	6.867238	8.724073
4.7	6.409872	7.11813	8.930748
4.8	6.690871	7.37891	9.146033
4.9	6.983326	7.650551	9.370625
5	7.288441	7.934115	9.605276

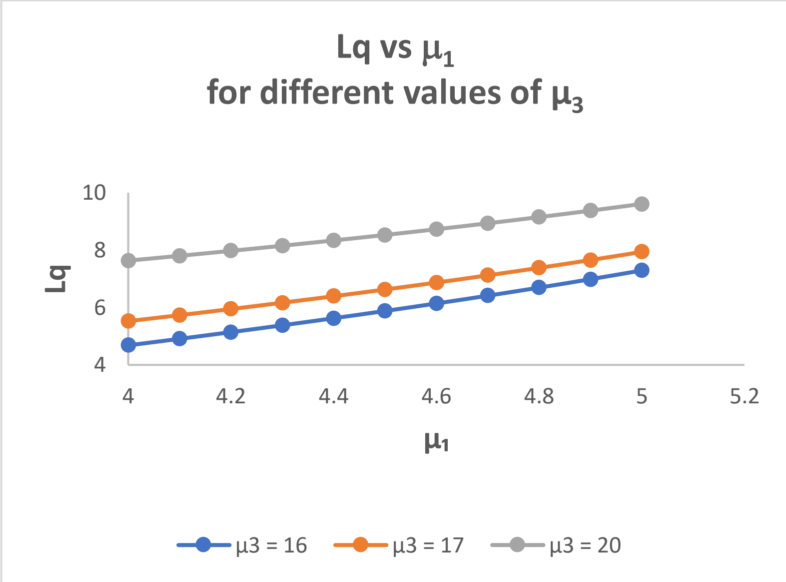

Fig. 4.6

Following can be interpreted from **Table 4.6** and **Fig. 4.6**:

(i) Mean queue length (L_q) increases with respect to the increase in first server service rate (μ_1).

(ii) Mean queue length (L_q) increases with the increase in service rate of third server (μ_3).

4.7. Behaviour of mean queue length of the system (L_q) with respect to service rate of first server (μ_1) for different values of fourth server service rate (μ_4) is depicted in Table 4.7 and in Fig. 4.7 keeping the values of other parameters as fixed.

Table 4.7

$\lambda=0.1$, $\mu_2= 2$, $\mu_3=16$, $A_{12}= 0.2$, $A_{13}=0.3$, $A_{14}=0.5$, $B_2=0.4$, $B_{21}=0.3$, $B_{23}=0.2$, $B_{24}=0.1$, $C_3=0.6$, $C_{31}=0.2$, $C_{32}=0.15$, $C_{34}=0.05$, $D_4=0.1$, $D_{41}=0.7$, $D_{42}=0.15$, $D_{43}=0.05$			
	$m_4 = 8$	**$m_4 = 8.5$**	**$m_4 = 9$**
μ_1	**Lq**	**Lq**	**Lq**
4	4.729041	4.680863	4.650658
4.1	4.994137	4.908278	4.847207
4.2	5.266029	5.140963	5.047918
4.3	5.545634	5.37962	5.25334
4.4	5.833938	5.625002	5.464057
4.5	6.132006	5.877914	5.680692
4.6	6.440994	6.139225	5.903911
4.7	6.762156	6.409872	6.134428
4.8	7.096867	6.690871	6.373013
4.9	7.44663	6.983326	6.620494
5	7.813098	7.288441	6.877771

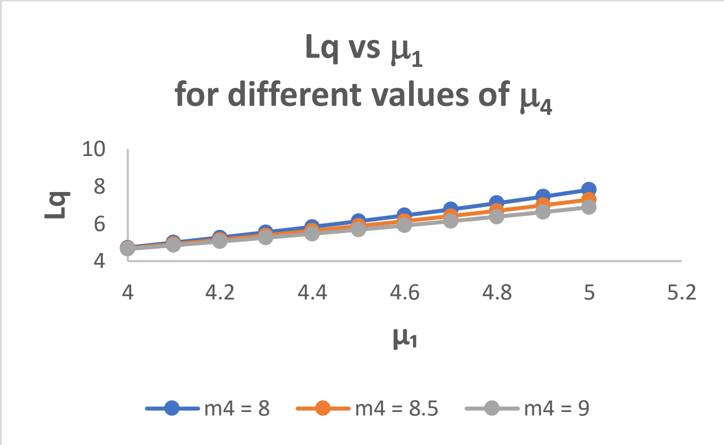

Fig. 4.7

Following can be interpreted from **Table 4.7** and **Fig. 4.7**:

(i) Mean queue length (L_q) increases with respect to the increase in first server service rate (μ_1).

(ii) Mean queue length (L_q) decreases with the increase in service rate of fourth server (μ_4).

4.8. Behaviour of mean queue length of the system (L_q) with respect to probability D_4 for different values of arrival rate (λ) is depicted in Table 4.8 and in Fig. 4.8 keeping the values of other parameters as fixed.

Table 4.8

μ_1=3, μ_2 = 4, μ_3= 5, μ_4= 0.2, A_{12}= 0.2, A_{13}=0.3, A_{14}=,0.5, B_2=0.4, B_{21}=0.3, B_{23}=0.2, B_{24}=0.1, C_3=0.6, C_{31}=0.2, C_{32}=0.15, C_{34}=0.05, D_4=0.1, D_{41}=0.7, D_{42}=0.15, D_{43}=0.05			
	λ=0.1	**λ=0.2**	**λ=0.3**
D_4	**Lq**	**Lq**	**Lq**
0.1	18.93478	41.75441	87.26598
0.15	17.21208	38.52471	80.39224
0.2	15.58603	35.51016	74.08713
0.25	14.04991	32.69285	68.29149
0.3	12.59756	30.05663	62.95362
0.35	11.22335	27.58698	58.02811
0.4	9.922103	25.27079	53.47501
0.45	8.689056	23.0962	49.25905
0.5	7.519831	21.05246	45.34896
0.55	6.410388	19.12979	41.71701
0.6	5.357002	17.31933	38.33848

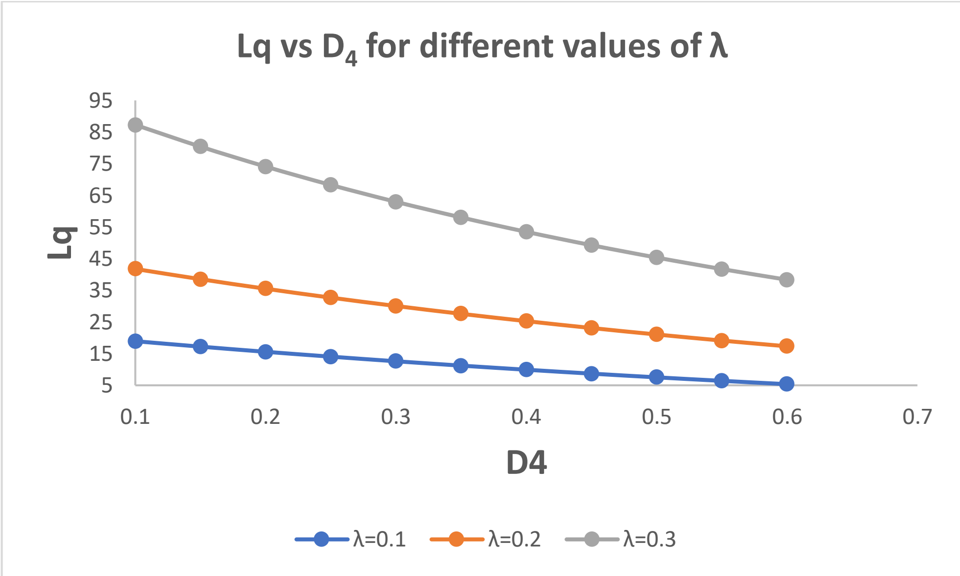

Fig. 4.8

Following can be interpreted from **Table 4.8** and **Fig. 4.8**:

(i) Mean queue length (L_q) decreases with respect to the increase in probability D_4.

(ii) Mean queue length (L_q) increases with the increase in arrival rate (**λ**).

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