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A Probabilistic Framework for Evaluating Efficiency Gains in Vogel's Approximation Method

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Abstract:

Transportation involves various uncertainties in determining the most efficient method for transporting goods. When a company has supply and destination points, or a supplier has specific demands, achieving an optimized balance becomes crucial. This task becomes more challenging when there is an imbalance between demand and supply, requiring an effective method to measure and address uncertainty. Probability theory provides a reliable framework for quantifying these uncertainties. Basic feasible solutions to transportation problems can be obtained using methods such as the North-West Corner Method, Least Cost Method, Row (Column) Minima Method, or Vogel's Approximation Method. Among these, Vogel's Approximation Method is widely recognized for its effectiveness in yielding superior results. This paper proposes a novel approach that integrates probability analysis into transportation problems, emphasizing the derivation of the probability distribution function (PDF) for Vogel's Approximation Method. The derived results not only enhance understanding but also offer valuable insights to improve decision-making processes within the transportation domain.

Keywords: Probability, Probability Distribution, Vogel's Approximation Method, Transportation Barrier, Uncertainty,Transportation Problem.

1. Introduction

In Operations Research (OR), specialized methods and techniques are employed to solve complex real-world problems. OR has widespread applications across various fields, particularly in the applied sciences, where it is used to tackle issues such as transportation optimization, queuing systems, game theory challenges, and job-machine assignment problems. These methodologies enable decision-makers to analyze and optimize systems to achieve the most efficient and cost-effective outcomes. The transportation problem, a key area in OR, focuses on optimizing the allocation of resources to meet supply and demand efficiently, making it a vital tool for industries aiming to streamline logistics.

In probability theory, the determination of probabilities for a defined problem can be approached in two primary ways: through events or probability distributions. Events must be clearly defined, as this clarity forms the foundation for calculating probabilities. Alternatively, probability distributions provide a systematic way to determine probabilities by selecting an appropriate distribution model. While numerous probability distributions exist, the choice of distribution depends on the specific characteristics of the problem. In transportation problems, identifying the appropriate distribution is crucial, as it allows for a more accurate representation of uncertainties and enhances the overall optimization process.

In the transportation problem, uncertainty in supply and demand at different locations can significantly impact the efficiency of logistics operations. The Poisson distribution offers a robust method to model this randomness, providing a probabilistic approach to account for variability in shipments. By integrating Poisson-distributed demands into transportation models, decision-makers can better handle uncertainties and achieve optimized resource allocation. This not only reduces transportation costs but also enhances the overall efficiency and reliability of logistics systems, making it a valuable tool for addressing real-world challenges in supply chain management.

While Operations Research (OR) focuses on applying practical techniques to solve complex optimization problems, Group Theory delves into the theoretical aspects of mathematical operations and their inherent properties. Group Theory provides a framework to study the underlying structures and behaviors of operations, which can complement OR by offering deeper insights into the mathematical principles driving optimization methods. Together, these fields bridge the gap between theory and application, enabling the development of more robust and efficient problem-solving approaches.

One widely used method in OR for solving transportation problems is Vogel's Approximation Method (VAM). VAM provides an efficient approach to finding an initial feasible solution that minimizes transportation costs. A fundamental concept in VAM is the calculation of "penalties," which represent the difference between the smallest and the second smallest cost elements in a row or column. By identifying and prioritizing the highest penalties, the method ensures that allocations are made in a way that maximizes cost-effectiveness. This systematic approach not only improves the solution process but also serves as a foundation for further optimization techniques, making VAM a cornerstone of transportation problem-solving strategies.

In Operations Research, Vogel's Approximation Method (VAM) stands out as a pivotal technique for addressing transportation problems by optimizing cost-effective allocations. The method leverages a "penalty" approach, where the difference between the smallest and the second smallest values in each row or column is calculated to prioritize allocations. This systematic process ensures that resources are distributed efficiently while minimizing overall transportation costs. Building on this foundation, our research proposes a probabilistic framework to evaluate the performance of VAM. By analyzing its effectiveness under varying scenarios, this framework aims to quantify the benefits of VAM and uncover potential areas for improvement, offering valuable insights into its application and adaptability.

The remainder of this paper is organized as follows: Section 2 provides a comprehensive Literature Review, highlighting previous studies and key advancements relevant to transportation problems and probabilistic frameworks. Section 3 details the Methodology adopted in this research, including the formulation of the probabilistic framework for Vogel's Approximation Method (VAM) and the steps involved in its implementation. Section 4 explores the Applications and Scope of the Study, demonstrating the practical utility and adaptability of the proposed approach across various scenarios in logistics and supply chain management. Finally, Section 5 concludes the paper by summarizing the findings, emphasizing the contributions, and suggesting potential directions for future research.

2. Literature Review

Sahito et al. [1] have introduced significant advancements in addressing the optimality of transportation problems (TP) through a modified approach to Vogel's Approximation Method using Statistical Techniques (MVOTPST). This method enhances the efficiency of the Initial Basic Feasible Solution (IBFS) compared to traditional methods such as the North-West Corner Method (NWCM), Least Cost Method (LCM), and the original Vogel's Approximation Method (VAM). The MVOTPST approach incorporates statistical calculations, using variances derived from the two smallest costs column-wise and the two largest costs row-wise as penalties, thereby improving cost-effectiveness. While VAM generally outperforms NWCM and LCM, Sahito et al. highlighted scenarios where these methods can surpass VAM. Their rigorous validation with multiple examples demonstrates that the IBFS obtained using MVOTPST is often absolutely optimal, close to optimal, or better than VAM, showcasing its potential to refine transportation problem-solving techniques and achieve near-optimal solutions.

Niluminda et al. [2] have proposed a novel algorithm that refines Vogel's Approximation Method to address both balanced and unbalanced transportation problems (TPs) more effectively. Their approach introduces additional constraints, such as considering the least cost per unit of transportation, to overcome the limitations of traditional methods and enhance accuracy in solving TPs. This algorithm demonstrates its effectiveness through numerical examples, delivering optimal or near-optimal solutions while outperforming existing methods in terms of solution quality and computational efficiency. Niluminda et al. emphasize the broad applicability of their approach across various forms of TPs, including balanced and unbalanced scenarios with diverse origins and destinations. Their findings suggest that this algorithm significantly improves transportation cost prediction and route optimization, offering a transformative tool for industries such as supply chain management, production, and logistics. By reducing transportation costs, increasing operational efficiency, and enhancing customer satisfaction, the proposed algorithm represents a substantial advancement in the study and application of transportation optimization.

Avetisyan et al. [3] present significant advancements in the study of simple Lie algebras, focusing on two primary areas. The first area explores the representation theory of simple Lie algebras, where the authors introduce new universal formulas within Vogel's universal description, uncovering additional properties of these formulas that contribute to a deeper understanding of their structure. The second area of their work applies Vogel's description to a physical theory, specifically in the formulation of refined Chern-Simons theories on S^3 for each of the simple gauge groups, including the exceptional ones. These developments not only enhance the theoretical framework of Lie algebras but also demonstrate the practical application of these mathematical advancements in understanding physical phenomena, broadening the scope of Vogel's description in both abstract mathematics and theoretical physics.

Putra et al. [4] applied the Modified Vogel's Approximation Method (MVAM) to optimize transportation costs at UD Aisyah Aby Jaya, focusing on minimizing shipping costs by analyzing demand, supply, and transportation costs. The results demonstrated that MVAM significantly reduced transportation costs, achieving a more efficient allocation of goods compared to traditional methods. The study found that MVAM provided closer-to-optimal solutions in less time, although it requires accurate data and complex calculations for maximum effectiveness. The authors recommended staff training, using specialized software to minimize errors, and regularly updating transportation cost data to ensure optimal results. They concluded that MVAM is an effective strategy for reducing operational costs and enhancing logistics efficiency, providing a valuable tool for companies aiming to optimize their distribution processes.

Utama et al. [5] investigated the application of transportation methods to optimize manpower distribution costs for aircraft maintenance, specifically focusing on the A04-Check of ATR 72-600 aircraft at Airline X. With limited manpower, the airline needed to allocate resources efficiently to maintenance centers at Soekarno-Hatta (CGK), Juanda (SUB), and Sultan Hasanuddin (UPG) airports. Using simulation data on work volume, manhours, and manpower requirements, the study applied three transportation methods: Northwest Corner Method, Least Cost Method, and Vogel's Approximation Method (VAM). The results revealed that VAM provided the most cost-efficient solution, reducing manpower distribution costs to 2884 USD (approximately Rp.42,129,616.20), compared to 4718 USD with the Northwest Corner Method and 3451 USD with the Least Cost Method. These findings highlight the effectiveness of VAM in minimizing operational costs and optimizing resource allocation for maintenance activities in the aviation sector.

Khudaverdian et al. [6] explored the integral representation of the universal volume function of compact simple Lie groups, revealing six analytic functions on CP^2 that transform as two triplets under permutations of Vogel's projective parameters. Instead of the anticipated invariance under parameter permutations, they uncovered a more intricate covariance. The study provides an analytical continuation of these functions and calculates their variation under parameter permutations, generalizing Kinkelin's reflection relation on Barnes' G(1+N) function for any simple Lie group and any point on Vogel's plane. This universal relation reflects asymmetry analogous to the $N \leftrightarrow -N$ transformation for SU(N) groups and coincides with permutation relations on the SU(N) line. The findings also extend to the universal partition function of Chern-Simons theory on a three-dimensional sphere, demonstrating effects similar to modular covariance in gauge theory partition functions under modular transformations of couplings.

Rueda et al. [7] investigated the applicability of Vogel's correlation for predicting production performance of fractured wells below bubble point pressure. Their study, based on extensive numerical simulations, revealed that Vogel's correlation significantly underestimates the production of fractured wells in this regime. Unlike Vogel's suggested 45% correction to the Absolute Open Flow (AOF), Rueda et al. proposed a more accurate correction factor of 22%, highlighting the potential for significant errors in production forecasts when using Vogel's method for fractured wells. Furthermore, the study emphasized the strong dependence of multiphase flow effects on fracture conductivity, with higher conductivity fractures exhibiting larger gas banks and consequently greater deviations from single-phase flow behavior.

Hamzeh et al. [8] investigated the heat transfer and flow characteristics of a third-grade non-Newtonian nanofluid through a porous medium in an annular geometry under the influence of a magnetic field. Their study employed three viscosity models: constant, Reynolds, and Vogel. The authors utilized the Akbari-Ganji's Method (AGM) to analytically solve the governing equations, demonstrating its high accuracy compared to numerical solutions. Key findings included the significant impact of pressure gradient, thermophoresis, and Brownian motion parameters on velocity and nanoparticle concentration. Furthermore, the study revealed that the Vogel's viscosity model yielded the highest velocity values. Importantly, Hamzeh et al. concluded that AGM provides an efficient and accurate approach for analyzing such complex fluid flow scenarios, offering valuable insights into the behavior of nanofluids in biomedical applications.

Caldwell et al. [9] recognized the limitations of the traditional Vogel's inflow performance relationship (IPR) when applied to undersaturated oil reservoirs. While Vogel's model assumes the initial reservoir pressure equals the bubble point pressure, which is inaccurate for undersaturated reservoirs. Caldwell et al. addressed this issue by modifying the Vogel IPR curve to accurately account for the presence of gas in the reservoir at initial conditions below the bubble point pressure. This modification improves the prediction of well performance in undersaturated reservoirs, leading to more accurate production forecasts and optimized field development plans.

Erza et al. [10] conducted a study at PT VictorindoKimiatama, a paint and coating distributor, to optimize their distribution channels and minimize costs. They compared the Vogel's Approximation Method (VAM) and the least cost method for calculating distribution costs. The study found that the least cost method resulted in lower total distribution costs compared to VAM, demonstrating its potential for cost savings in the company's operations. This research highlights the importance of employing appropriate transportation models for optimizing distribution networks and improving overall operational efficiency.

3. Methodology

Let us consider Transportation Problem shown in Table 1 in context of cost as below

Table 1: Transportation Problem

From ↓	Destination	Destination	…..	Destination	Supply

To →	D_1	D_2		D_n	
O_1	e_{11}	e_{12}	$\dots$	e_{1n}	s_1
O_2	e_{21}	e_{22}	$\dots$	e_{2n}	s_2
:	:	:	:	:	:
O_m	e_{m1}	e_{m2}	$\dots$	e_{mn}	s_m
Demand	d_1	d_2	$\dots$	d_n	Balanced i.e. $\sum_{i=1}^{m} s_i = \sum_{j=1}^{m} d_j$

3.1 Probabilistic Framework and Poisson Distribution in context of Transportation Problem

We can use a probabilistic framework to represent the costs and allocations. Here's a step-by-step approach to creating a probability equation for the transportation problem:

Step 1: Define A Variable and Parameters

Decision Variable

x_{ij} Amount shipped from origin O_i to destination D_j

Parameters

e_{ij} Unit Cost of Shipping from Origin O_i To Destination D_j

s_i Supply at Origin O_i

d_j Demand at Destination D_j

Balanced Condition

The problem is balanced. i.e. $\sum_{i=1}^{m} s_i = \sum_{j=1}^{m} d_j$

Step 2: Probabilistic Representation

Cost Matrix

Let us consider $E = [e_{ij}]$ is cost matrix.

Supply and Demand as Random Variables

Assume that supplies s_i and demands d_j are random variables characterized by discrete probability distributions.

Step 3: Define the Discrete Probability Distribution

Let the costs e_{ij} be considered as random variables that follow a discrete probability distribution. We can use a Poisson distribution to model the allocations x_{ij}.

Probability of Allocation

$P(X_{ij}=x_{ij})$ represents the probability that x_{ij} units are shipped from O_i to D_j.

Constraints

$$\sum_{j=1}^{m} x_{ij} = s_i; foreachoriginO_i$$
$$\sum_{i=1}^{n} x_{ij} = d_j; foreachdestinationD_j$$

Since the problem involves random variables representing discrete quantities of shipments, a Poisson distribution can be appropriate:

$$X_{ij} \sim Poission(\lambda_{ij})$$

Here, λ_{ij} is rate parameter of poission distribution.

Step 4: Construct the Probability Equation

For constructing a probability equation, we consider the joint probability distribution of all allocation. Given the constraints, we can write the joint probability as:

$$P(X_{11} = x_{11}, X_{12} = x_{12}, \dots, X_{mn} = x_{mn}) = \prod_{i=1}^{m} \prod_{j=1}^{n} P(X_{ij} = x_{ij})$$

Assuming that the allocations $x_{ij}$are independent given the supplies and demands, the probability of a particular allocation x_{ij} can be modeled using a Poisson distribution:

$$P\left(X_{ij} = x_{ij}\right) = \frac{e^{-\lambda_{ij}} (\lambda_{ij})^{x_{ij}}}{x_{ij}!}; \ \lambda_{ij} = \frac{s_i d_j}{\sum_{i=1}^{m} \sum_{j=1}^{n} s_i d_j} \tag{1}$$

Step 5: To Obtain Objective Function

To construct or obtain the objective function, we aim to minimize the total transportation cost.

From the above, we get total transportation cost $\sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} X_{ij}$. To minimize this total transportation cost, we consider its expected value. The expected total transportation cost is

$$E\left(\sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} X_{ij}\right) = \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} E(X_{ij}) \tag{2}$$

(From the linearity property of expectation)

We know that expected value of object in multinomial distribution is it’s parameter. From this, we get $E\left(X_{ij}\right) = \lambda_{ij}$. Now, substituting the value of $E\left(X_{ij}\right)$ and From equation (1) and (2), we get

$$E\left(\sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} X_{ij}\right) = \sum_{i=1}^{m} \sum_{j=1}^{n} e_{ij} \frac{s_i d_j}{\sum_{i=1}^{m} \sum_{j=1}^{n} s_i d_j} \tag{3}$$

Therefore, the objective function to minimize the expected total transportation cost is:

$$\boldsymbol{Minimize} \sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{e_{ij}} \frac{\boldsymbol{s_i d_j}}{\sum_{i=1}^{m}\sum_{j=1}^{n} \boldsymbol{s_i d_j}}$$

This probabilistic framework models the transportation problem by considering the costs and allocations as random variables following a Poisson distribution.

3.1.1 Characteristic Function

For a Poisson distribution X_{ij} with parameter λ_{ij}, the characteristic function $\phi_{X_{ij}}(t)$ is given by:

$$\emptyset_{X_{ij}}(t) = E\left(e^{itX_{ij}}\right) = \exp(\lambda_{ij}\left(e^{it} - 1\right))$$

This is useful for understanding the distribution's properties and for finding the distribution of sums of independent random variables.

Similarly, we get Characteristic Function for Total Transportation Cost C is,

$$\emptyset_C(t) = \exp(\sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij} \frac{s_i d_j}{\sum_{i=1}^{m}\sum_{j=1}^{n} s_i d_j}\left(e^{ite_{ij}} - 1\right))$$

This characteristic function provides a thorough description of the statistical behavior of the total cost in the probabilistic transportation problem by summarizing its distributional parts.

3.1.2 Moment Generating Function (MGF)

The MGF $M_{X_{ij}}(t)$ for a Poisson random variable X_{ij} is:

$$M_{X_{ij}}(t) = E(e^{tX_{ij}}) = \exp(\lambda_{ij}(e^{t} - 1))$$

This facilitates the computation of moments (mean, variance) and is useful for deriving properties of the distribution.

3.1.3 Variance

The variance of a Poisson random variable X_{ij} is equal to its parameter:

$$Var(X_{ij}) = \lambda_{ij}$$

This provides a measure of the dispersion or variability of the shipment amounts from their expected values

3.1.4 Co-Variance

For Independent Poisson random variables X_{ij} and X_{kl} (where $(i, j) \neq (k, l)$):

$$Cov(X_{ij}, X_{kl}) = 0$$

If X_{ij} and X_{kl} are not independent, the covariance can be calculated based on their joint distribution.

This helps understand the relationship between different shipment amounts and their dependency, which is critical for optimizing transportation costs.

3.2 The Multinominal Distribution in context of Transportation Problem

Step 1,2, and 3 should be followed as shown in section 3.1

Step 4: Define the Multinomial Distribution and Probability Allocation

Let us assume that the allocation from each origin O_i to the destinations follows a multinomial distribution. The probabilities p_{ij} represent the fraction of supply s_ii that goes to destination D_j.

$$(X_{i1}, X_{i2}, \dots, X_{in}) \sim Multinomial(s_i, (\, p_{i1}, p_{i2}, \dots, p_{in}))$$

Thus, $P((X_{i1} = x_{i1}, X_{i2} = x_{i2}, \dots, X_{in} = x_{in}) = \frac{s_i!}{x_{i1}!x_{i2}!\dots x_{in}!} p_{i1}{}^{x_{i1}} p_{i2}{}^{x_{i2}} \dots p_{in}{}^{x_{in}}$

Here The probabilities p_{ij} are defined based on the demand d_j and the total supply:

$$p_{ij} = \frac{d_j}{\sum_{k=1}^{n} d_k} \tag{4}$$

Step 5: Finding Objective Function

The objective is to minimize the expected total transportation cost:

$$Minimize E(\sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij}X_{ij}) = \sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij}E(X_{ij})$$

For Multinomial random variable X_{ij}, the expected value $E(X_{ij})$ is:

$$E(X_{ij}) = s_i p_{ij} = s_i \frac{d_j}{\sum_{k=1}^{n} d_k} \qquad \text{(From Equation (4))}$$

Thus, Objective Function becomes:

$$Minimize \sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij} s_i \frac{d_j}{\sum_{k=1}^{n} d_k}$$

3.2.1 Moment Generating Function (MGF)

The moment-generating function (MGF) for a multinomial random vector $(X_{i1}, X_{i2}, \dots, X_{in})$ is given by:

$$M_X(t) = E(e^{tX})$$

For multinomial random vector $(X_{i1}, X_{i2}, \dots, X_{in})$ with parameter n (total supply) and

$p = (\, p_{i1}, p_{i2}, \dots, p_{in})$ (probabilities), the MGF is given by

$$M_X(t) = E(e^{tX}) = \left(\sum_{j=1}^{n} p_{ij} e^{t_j}\right)^{s_i}$$

Where $t = (t_1, t_2, \dots, t_n)$

This function aids in the computation of moments of distribution which are essential for comprehending the distribution and central tendencies of shipment quantities.

3.2.2 Characteristic Function

For the multinomial distribution, the characteristic function can be derived similarly to the MGF by replacing t_j with it_j.

The Characteristic Function is

$$\emptyset_X(t) = E\left(e^{itX}\right) = \left(\sum_{j=1}^{n} p_{ij} e^{it_j}\right)^{s_i}$$

This function helps with the analysis of the distributions properties by transforming the problem into the frequency domain.

3.2.3 Variance

The variance of these shipments, representing the variability around the mean, is:

$$Var\left(X_{ij}\right) = -s_i p_{ij}(1 - p_{ij})$$

This variance quantifies the uncertainty in the shipment quantities due to demand fluctuations and allocation strategies.

3.2.4 Co-Variance

The covariance between X_{ij} and X_{ik} for $j \neq k$ is:

$$Cov\left(X_{ij}, X_{ik}\right) = -s_i p_{ij} p_{ik}$$

This negative covariance shows that given the fixed total supply an increase in shipments to one destination typically leads to a decrease in shipments to another.

Table 2: Probability Approach in Vogel's Approximation Method

Sr. No.	Events	Description of the Events	Possible Outcome	Probability
1	E_{1j}	The event of allocation among $(1,j)^{th}$ cells	$\{d_j\} or \{s_1\} or \{s_1 - d_j\}$	$P(E_{1j}) = \begin{cases} 1 & ; if E_{1j} = \{d_j\} or E_{1j} = \{s_1\} \\ \frac{1}{n(\{s_1 - d_j\})} & ; if E_{1j} = \{s_1 - d_j\} \\ 0 & ; Otherwise \end{cases}$
2	E_{i1}	The event of allocation among $(i,1)^{th}$ cells	$\{d_1\} or \{s_i\} or \{d_1 - s_i\}$	$P(E_{i1}) = \begin{cases} 1 & ; if E_{i'j} = \{d_j\} or E_{i'j} = \{s_1\} \\ \frac{1}{n(\{d_1 - s_i\})} & ; if E_{i'j} = \{d_1 - s_i\} \\ 0 & ; Otherwise \end{cases}$
3	$E_{i^*j'}$	The event of allocation in a least cost cell in Transportation Table	$\{d_{j'}\} or \{s_{i^*}\} or \{d_{j'} - s_{i^*}\} or \{s_{i^*} - d_{j'}\}$	$P(E_{i^*j'}) = \begin{cases} 1 & ; if E_{i^*j'} = \{d_{j'}\} or E_{i'j} = \{s_{i^*}\} \\ \frac{1}{n(\{d_{j'} - s_{i^*}\})} & ; if E_{i^*j'} = \{d_{j'} - s_{i^*}\} \\ \frac{1}{n(\{s_{i^*} - d_{j'}\})} & ; if E_{i^*j'} = \{s_{i^*} - d_{j'}\} \\ 0 & ; Otherwise \end{cases}$

3.3 Results and Proofs

Result 1: If $\boldsymbol{e_{ij}}$ is a least cost in Transportation Table and demand $\boldsymbol{d_j}$ is not allocated in respective column or some demand from $\boldsymbol{d_j}$ is remain to allocate then Probability of allocation in $(\boldsymbol{i},\boldsymbol{j})^{\boldsymbol{th}}$ cell is 1 using Vogel's Approximation Method.

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Proof: Let us consider $\mathrm{e_{ij}}$ is a least cost in Transportation Table.

$$\text{i.e. } \min\left(\mathrm{e_{11}}, \ldots., \mathrm{e_{1n,}} \mathrm{e_{21}}, \ldots, \mathrm{e_{2n,}} \ldots.., \mathrm{e_{m1}}, \ldots.., \mathrm{e_{mn}}\right) = \mathrm{e_{ij}}; \text{i and j is fix.}$$

We are supposing that demand d_j is not allocated in respective column or some demand from d_j and $\mathrm{E_{ij}}$is the event of allocation in a least cost cell in Transportation Table.

In Vogel's Approximation Method, Penalty is the value of difference between two least costs in the row or column.

Suppose Penalty of either ith row or jth column is highest in 1st Phase.

$$\text{If Penalty of ith row is highest then } \min(\mathrm{e_{i1}}, \ldots., \mathrm{e_{in}}\ ) = \mathrm{e_{ij}}$$

Using Vogel's Approximation Method,

We should allocate minimum cost according to require supply and available demand.

At ith row, require supply is $\mathrm{s_i}$ and available demand is $\mathrm{d_j}$.

Now,

$$\text{Total cases for allocation are } \{\mathrm{d_j}\} \text{ or } \{\mathrm{s_i}\} \text{ or } \left\{\mathrm{d_j} - \mathrm{s_i}\right\} \text{ or } \left\{\mathrm{s_i} - \mathrm{d_j}\right\}.$$

As per our assumption, $\mathrm{d_j}$ is not allocated in respective column or some demand from $\mathrm{d_j}$ is remain to allocate. So,

$$\mathrm{P}(\left\{\mathrm{d_j} - \mathrm{s_i}\right\}) = 0 \text{ and } \mathrm{P}\left(\left\{\mathrm{s_i} - \mathrm{d_j}\right\}\right) = 0$$

Using above probability table, we get

$$\mathrm{P}(\mathrm{E_{ij}}) = 1$$

Hence, Ife_{ij} is a least cost in Transportation Table and demand d_j is not allocated in respective column or some demand from d_j is remain to allocate then Probability of allocation in $(i, j)^{th}$ cell is 1 using Vogel's Approximation Method.

Result 2: The Unit Cost for first allocation in below transportation problem is always 0 using Vogel's Approximation Method or Least Cost Method.

Table 3: Transportation Problem

From ↓ To →	Destination D1	Destination D2	…..	Destination Dn	Supply
O1	0	e_{12}	…	e_{1n}	s_1
O2	e_{21}	0	…	e_{2n}	s_2
:	:	:	:	:	:
On	e_{n1}	e_{n2}	…	0	s_n
Demand	d_1	d_2	…	d_n	Balanced i.e. $\sum_{i=1}^{n} s_i = \sum_{j=1}^{n} d_j$

Where $O_i = D_j\, ; Foralli = j$

Proof: Let us consider the above Transportation Problem (TP).

We know

$$O_i = D_j\, ; Foralli = j$$

That means each origin is also in place of destination respective number.

During finding penalty of each column, we have smallest element 'o' and we have to find nearest smallest element. Similarly for row. Whatever nearest smallest element in each column or row is to be a penalty of respective column or row.

Suppose maximum penalty among row penalties and column penalties is e_{ij}.

Now, we should find minimum element in respective row or column.

Table 3: Special Case of Transportation Problem

From ↓	Destination D1	Destination D2	…..	Destination Dn	Supply

To →					
O1	0	e_{12}	…	e_{1n}	s_1
O2	e_{21}	0	…	e_{2n}	s_2
:	:	:	:	:	:
Om	e_{m1}	e_{m2}	…	0	s_m
Demand	d_1	d_2		d_n	Balanced i.e. $\sum_{i=1}^{m} s_i = \sum_{j=1}^{m} d_j$

Where $O_i = D_j$; $For alli = j$

As above problem, Unit cost is never negative and each row or column has at least one 'zero'. So, it is to be smallest element.

According to VAM, we should be allocated at '0' Unit cost in first iteration.

Hence the proof.

4. Applications and Scope

4.1 Applications of the Study

This research significantly advances the allocation process in transportation problems, ensuring optimal resource distribution that enhances efficiency and minimizes costs. The study establishes a probabilistic framework that deepens our understanding of the allocation process, leading to more reliable and effective solutions for real-world applications. By incorporating the randomness in supplies, demands, and transportation costs, this framework provides a comprehensive probabilistic analysis, addressing the inherent uncertainties in transportation logistics.

The probabilistic insights gained from this research offer practical value across multiple domains, improving decision-making by enabling data-driven, optimized solutions. Specifically, the study uses Poisson distribution to model the likelihood of delivering specific quantities between locations, helping to overcome the challenge of uncertainty in supply and demand. The probability equation developed in this study aids in assessing efficiency gains by evaluating the expected total transportation cost, offering a valuable tool for identifying the most cost-effective strategies. Ultimately, the approach enhances the ability to develop efficient delivery plans and optimize transportation logistics, even when exact supplies and demands are unpredictable. This framework holds potential for transforming decision-making processes in transportation and operational research.

4.2 Scope of the Study

This study focuses on evaluating probabilistic outcomes in transportation problems through the application of Vogel's Approximation Method (VAM). It emphasizes defining allocation events and analyzing their associated probabilities, with the goal of enhancing the understanding of VAM's efficiency in resource allocation. Additionally, the research explores its practical applicability in optimizing transportation logistics, providing valuable insights for improving operational effectiveness.

5. Conclusions

Based on the analysis of Vogel's Approximation Method (VAM) for solving transportation problems, it can be concluded that this approach provides a robust probabilistic framework for optimizing allocation decisions. By utilizing penalties derived from the differences in transportation costs, VAM effectively balances supply and demand across multiple sources and destinations. Its iterative methodology ensures that allocations are executed with minimal cost implications, thereby enhancing operational efficiency in logistics and supply chain management.

Moreover, VAM consistently delivers near-optimal solutions, demonstrating its reliability even in dynamic supply and demand scenarios. The probabilistic foundation of the method facilitates a systematic allocation process, ensuring maximum resource utilization while adhering to logistical constraints. This adaptability makes it particularly effective in addressing real-world transportation challenges.

In summary, Vogel's Approximation Method serves as a powerful tool for tackling transportation problems, offering significant efficiency gains and cost reductions in complex supply chain operations. Its versatility and applicability across various industries highlight its value as a strategic decision-making instrument for optimizing logistical processes and resource allocation.

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Conflict Statement

There is no conflict of interest.

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